Erikson Institute
Early Math Collaborative

Using *NUMBER* Sense
to Build *OPERATIONS* Sense

Lisa Ferguson
Instructional Coach & PD Facilitator

5-8 Breakout Session, *Children as Mathematicians* PD Institute
Omaha, Nebraska
January 25 & 27, 2018

Let's Do Some Math!

A Tale of Feet & Tails

My Grandpa is a funny guy. He always tells people, "When I look around my house, I can count some feet and some tails."

Let's Do Some Math!

A Tale of Feet & Tails

My Grandpa is a funny guy. He always tells people, "When I look around my house, I can count some feet and some tails."

What is going on?
What do you "see"?
What do we know?

Let's Do Some Math!

A Tale of Feet & Tails

My Grandpa is a funny guy. He always tells people, "When I look around my house, I can count 14 feet and 2 tails."

Let's Do Some Math!

A Tale of Feet & Tails

My Grandpa is a funny guy. He always tells people, "When I look around my house, I can count 14 feet and 2 tails."

What are the quantities?
What do they mean?
How are they related?

Let's Do Some Math!

A Tale of Feet & Tails

My Grandpa is a funny guy. He always tells people, "When I look around my house, I can count 14 feet and 2 tails."
Who lives in Grandpa's house?

Let's Do Some Math!

A Tale of Feet & Tails

My Grandpa is a funny guy. He always tells people, "When I look around my house, I can count 14 feet and 2 tails."
Who lives in Grandpa's house?

Work this out individually.

Let's Do Some Math!

A Tale of Feet & Tails

My Grandpa is a funny guy. He always tells people, "When I look around my house, I can count 14 feet and 2 tails."
Who lives in Grandpa's house?

Work this out individually.

Turn & Talk: Introduce yourself to a partner and share your results.

Continuing the Tale of Feet and Tails…

What did you learn about:
- solving problems?
- numbers and operations?

What if the numbers of feet and tails
were different?

Turn & Talk

- What did you notice about how this word problem was presented?

- How does this compare to what you usually do to engage students in a word problem?

Strategies Observed

- Focused on the "story"
- Delayed the mention of the quantities
- The question was provided as the *last* thing
- The operation needed to solve the problem was not mentioned
- No strategy was provided
- The story was read aloud

The "Three Reads" Strategy:

Making **SENSE**
of Problems
BEFORE
Solving Them

The "Three Reads" Strategy
OUTLINE

1. Comprehending the TEXT

The "Three Reads" Strategy
OUTLINE

1. Comprehending the Text
2. Comprehending the MATHEMATICS

The "Three Reads" Strategy
OUTLINE

1. Comprehending the Text
2. Comprehending the Mathematics
3. Eliciting Mathematical QUESTIONS

What are operations?
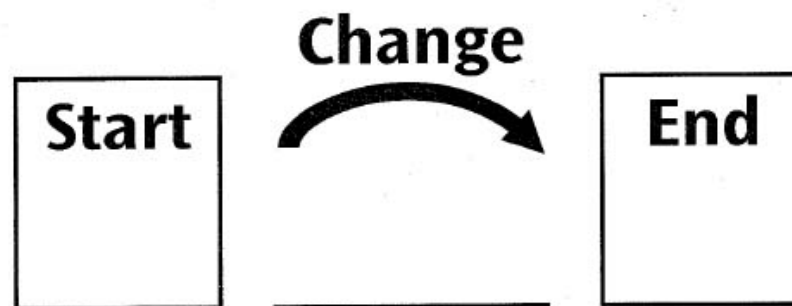
What are operations?

All operations tell a **STORY**.

Basic Types of
Addition & Subtraction Number Stories

- Change Situations
  - Join
  - Separate

- Relationships
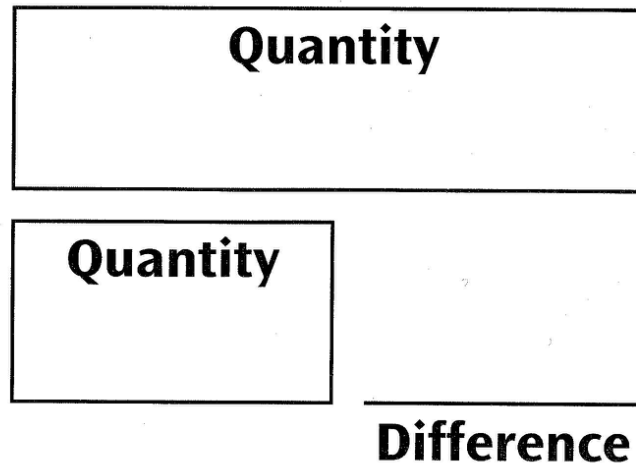 - Compare
  - Number Composition

By varying the unknown within each type of number story,
many different problem situations can be constructed.
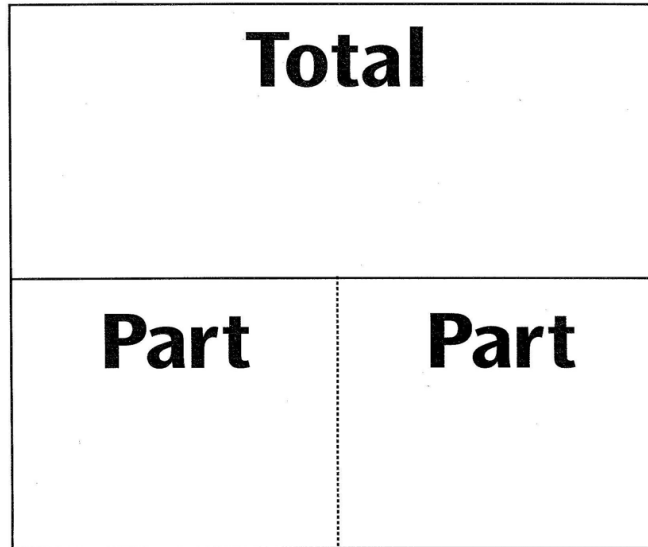
Change Situation: Joining or Separating

- Result Unknown - *How many in the end?*
- Change Unknown - *How many were added   or taken away?*
- Start Unknown - *How many were there at first?*

Relationship Situation: Comparison

- Difference Unknown - *How many more or less?*
- Quantity Unknown - *How many in the set?*

Relationship Situation: Number Composition

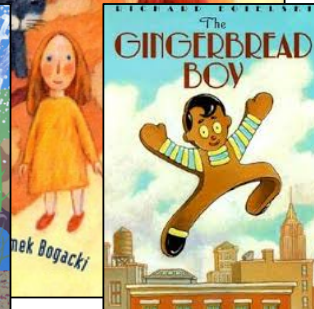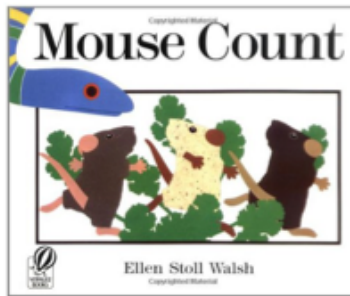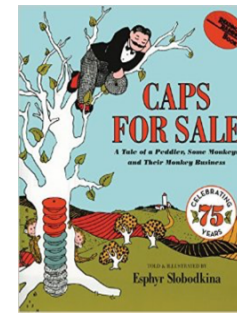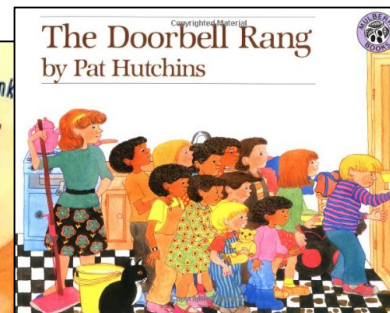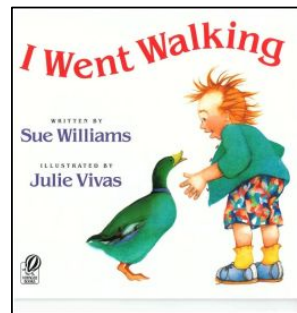| Total	
Part	**Part**

- Whole Unknown - *How many altogether?*
- Part Unknown - *How many to complete the whole?*

Big Ideas of Number Operations

Topic	Big Ideas	Examples
Changing Sets ⭐⭐ + ⭐⭐⭐	● Sets can be *changed* by adding items (joining) or by taking some away (separating).	• *You have 2 balls and I have 3 balls. How many balls do we have altogether?* ● *You had 12 cards, and you gave your friend 5. How many do you have now?*
Comparing Sets ⭐⭐ < ⭐⭐⭐	● Sets can be *compared* using the attribute of numerosity, and *ordered* by more than, less than and equal to.	● *I have a handful of raisins; Chris has a bowl-ful. Chris has more!* ● *I have 1 pear and 1 peach; you have 2 apples. We have the same number of fruits.* ● *Avery has 3 dirty plates, and Tracy has 4 dirty bowls. Who has fewer dishes to wash?*
Number Composition ⭐⭐ ⭐⭐	● A quantity (whole) can be *decomposed* into equal or unequal parts; the parts can be *composed* to form the whole.	• *How many ways can you show 5 with fingers on both hands?*

erikson
early math collaborative

Books can inspire number stories!

Learning Trajectory:
Development of Children's Solution Strategies

o Direct Modeling Strategies

   o Use of objects (manipulatives), fingers, or drawings to directly model the action or relationship described in the problem

Learning Trajectory:
Development of Children's Solution Strategies

o Direct Modeling Strategies
- o Use of objects (manipulatives), fingers, or drawings to directly model the action or relationship described in the problem

o Counting Strategies Use the counting sequence itself to figure out the solution
- o Usually involves a strategy to keep track of counts

video (counting on fro doubles): http://factfluency.com/Addition_Strategies.html

Learning Trajectory:
Development of Children's Solution Strategies

○ Direct Modeling Strategies

- ○ Use of objects (manipulatives), fingers, or drawings to directly model the action or relationship described in the problem

○ Counting Strategies Use the counting sequence itself to figure out the solution

- ○ Usually involves a strategy to keep track of counts

○ Derived Facts Strategies Use "friendly" numbers first: doubles, sums of tens

- ○ Over time, children learn many number facts at recall level

CGI Progression

Automaticity

Facts

Derived
Facts

Counting

Direct
Modeling

Early Numeracy
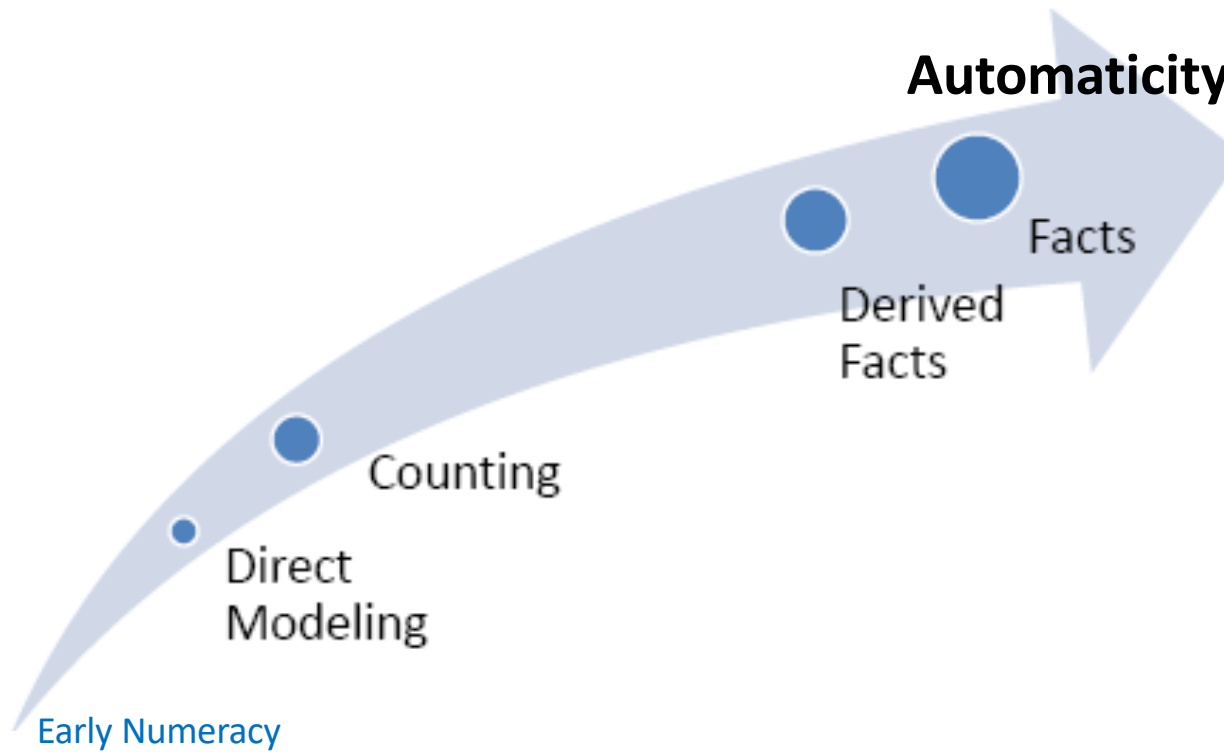
How do we build automaticity?

- Build Number Sense

" ...good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms."
(Howden)

CGI Progression

Automaticity

4 Relationships

Facts

Derived
Facts

Counting

Direct
Modeling

Early Numeracy

Four Relationships

- Spatial
- One & Two More, One & Two Less
- Benchmarks 5 and 10
- Part-Part Whole
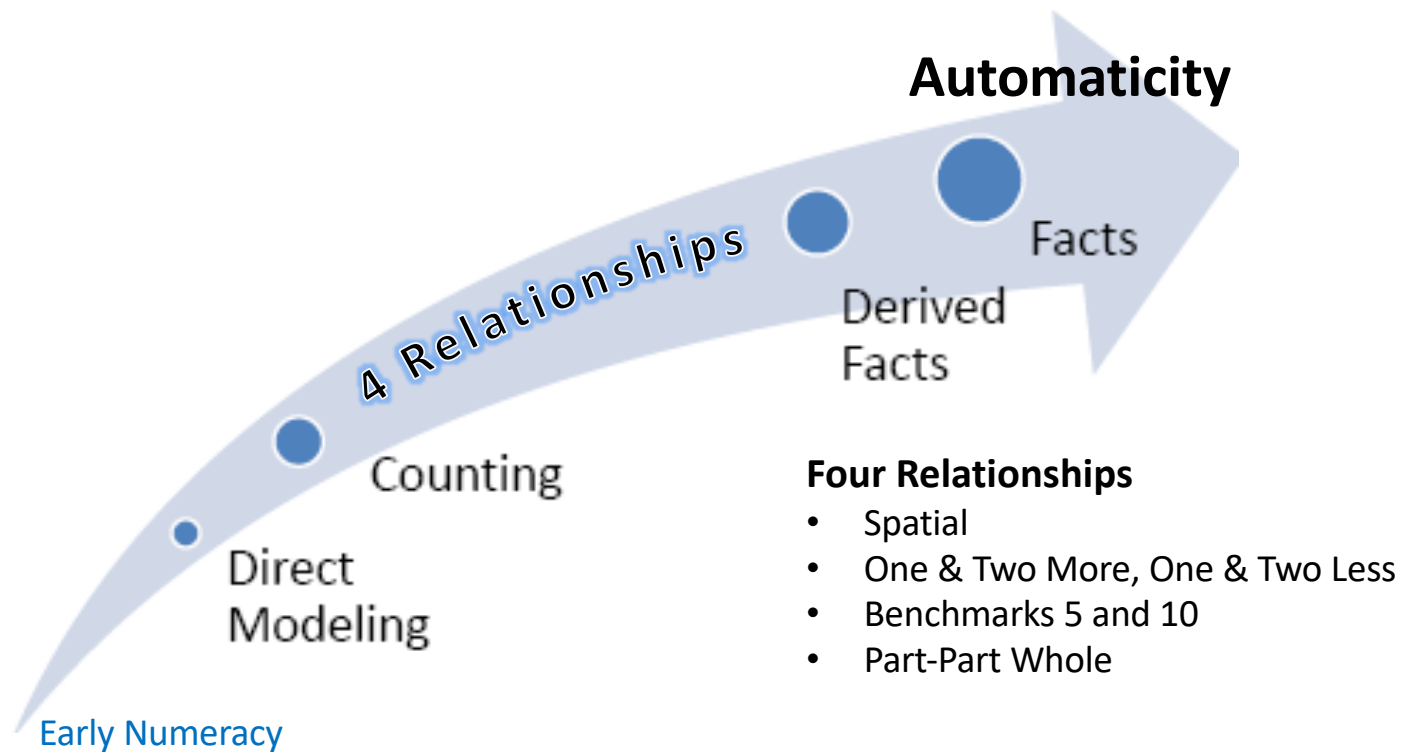
Four relationships that build number sense

- **Spatial Relationships** – recognizing how many without counting by seeing the visual pattern.

- **One & Two More, One & Two Less** – this is not the ability to count on two or count back two, but instead knowing which numbers are one more or two less than any given number.

- **Benchmarks of 5 and 10** – since 10 plays such an important role in our number system (and two 5s make up 10), students must know how numbers relate to 5 and 10.

- **Part-Part-Whole** – to conceptualize a number as being made up of two or more parts is the most important relationship to develop.

Van De Walle, 2006

Cognitively Guided Instruction Progression

6 + 7

- **Direct Modeler** – counts out 6 things, counts out 7 things, pushes them all together and counts the total.

- **Counter** – holds 6 in their head and counts on 7 more.

- **Derived Fact** – uses a fact they know to help them.

Derived Facts

6 + 7

- What are some derived facts that kids might use to solve this problem?

Use Number Sense to Help

6 + 7

6+6+1   6+4+3   7+7-1   3+3+7   1+5+5+2

Use Number Sense to Help

6 + 7

6+6+1   6+4+3   7+7-1   3+3+7   1+5+5+2

- **What relationships would students need to have before they can use the derived facts for this problem?**

Use Number Sense to Help

6 + 7

6+6+1   6+4+3   7+7-1   3+3+7   1+5+5+2

What relationships would students need to have before they can use the derived facts for this problem?

* spatial   * one/two more and less
* benchmarks of 5 & 10   * part-part-whole

When we focus on *NUMBER RELATIONSHIPS*,

• **children become "fluent" in their thinking about basic facts**

• **children's knowledge is extended when they encounter a new task**

When we build a child's *NUMBER SENSE,*
it promotes **THINKING** instead of just **computing**

Components of Fluency
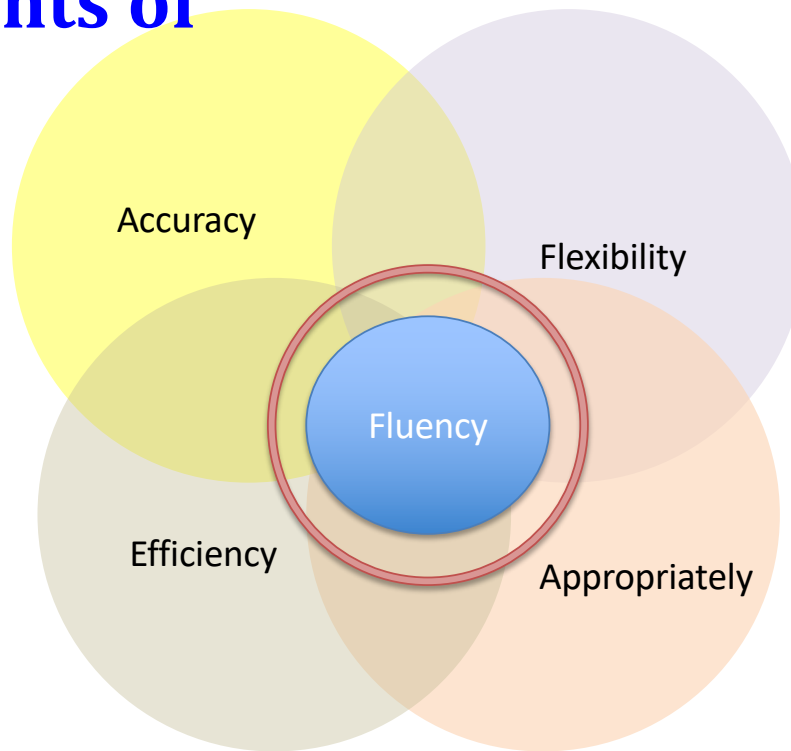
What is fluency?

❑ Accuracy – the ability to solve the problem correctly

❑ Efficiency – the ability to solve the problem in a
 • reasonable amount of time

❑ Flexibility – the ability to use a variety of strategies
 • to solve a particular problem (to be able to think about a
 • solution in a number of different ways)

❑ Appropriately – the ability to choosing the BEST
 • strategy for that particular problem and set of numbers

Lighten the Load – think relationships!

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

Purple – plus zero   **Green – facts that make 10**

Orange – doubles   **Blue – ten plus something**

Erikson Institute

Early Math Collaborative

100 Facts!

×	1	2	3	4	5	6	7	8	9	10
1	1×1	1×2	1×3	1×4	1×5	1×6	1×7	1×8	1×9	1×10
2	2×1	2×2	2×3	2×4	2×5	2×6	2×7	2×8	2×9	2×10
3	3×1	3×2	3×3	3×4	3×5	3×6	3×7	3×8	3×9	3×10
4	4×1	4×2	4×3	4×4	4×5	4×6	4×7	4×8	4×9	4×10
5	5×1	5×2	5×3	5×4	5×5	5×6	5×7	5×8	5×9	5×10
6	6×1	6×2	6×3	6×4	6×5	6×6	6×7	6×8	6×9	6×10
7	7×1	7×2	7×3	7×4	7×5	7×6	7×7	7×8	7×9	7×10
8	8×1	8×2	8×3	8×4	8×5	8×6	8×7	8×8	8×9	8×10
9	9×1	9×2	9×3	9×4	9×5	9×6	9×7	9×8	9×9	9×10
10	10×1	10×2	10×3	10×4	10×5	10×6	10×7	10×8	10×9	10×10

Ones (identity)

Doubles

Fours (double doubles)

Tens

Fives

Nines

Commutative

Erikson Institute

Early Math Collaborative

video analysis – focus on algorithm
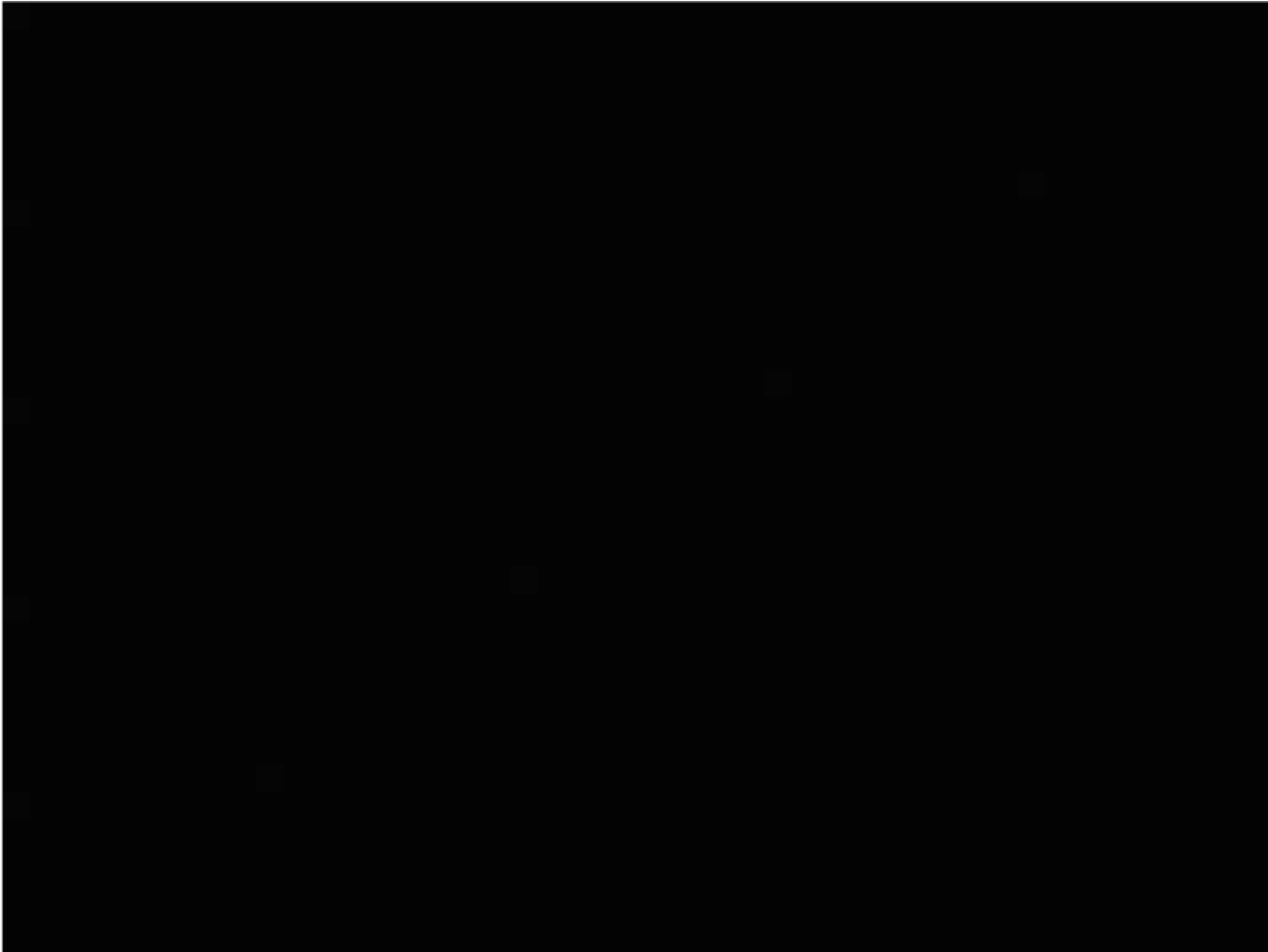
Video Analysis: *Focus On Number Relationships*
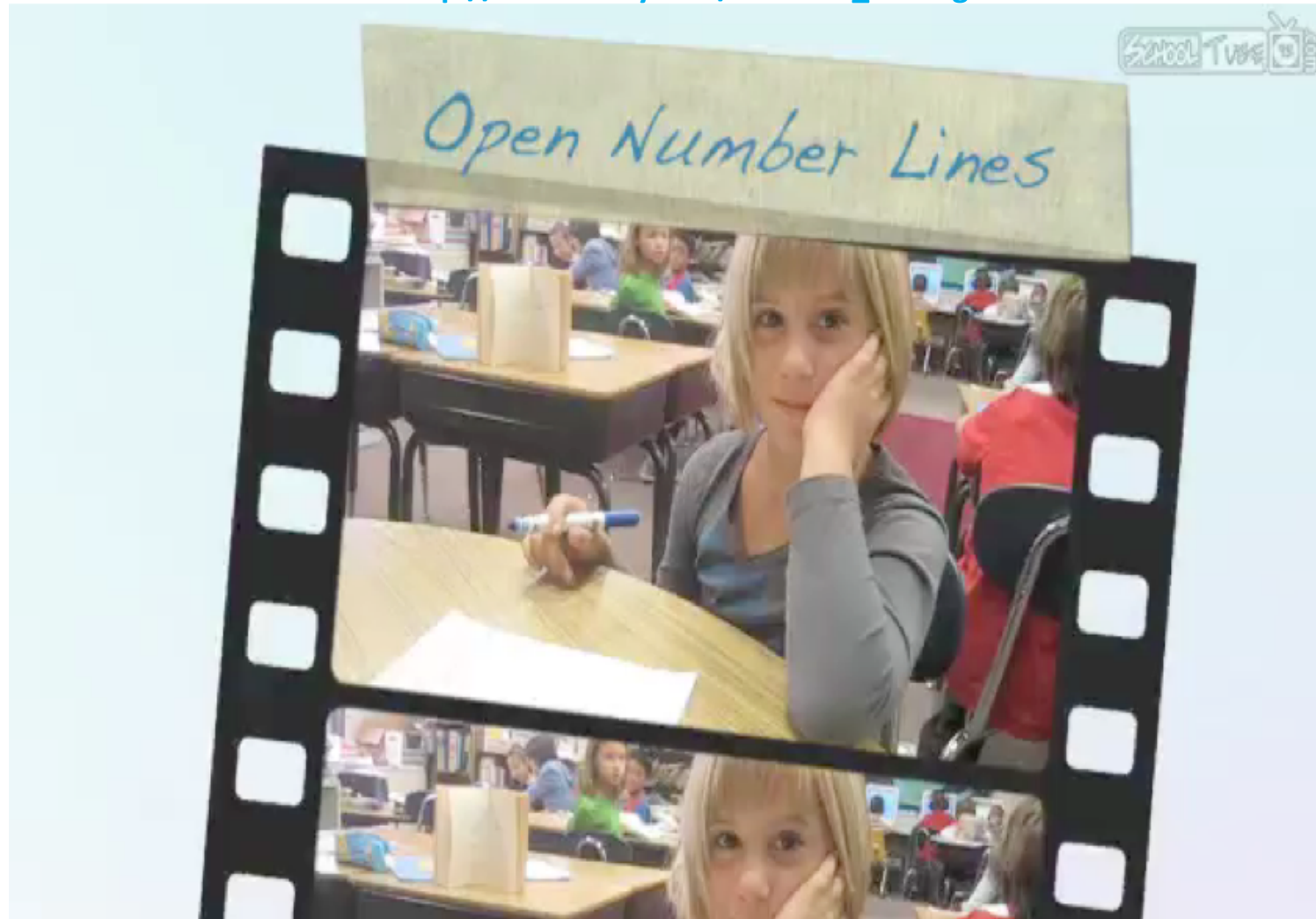
Combining the **4 NUMBER RELATIONSHIPS** with the
"THREE READS" STRATEGY can be

POWERFUL

for building good **OPERATIONS SENSE!**

It's all about the QUANTITIES!

ADJUSTING QUANTITIES
in story problems can give students opportunities to practice the
Number Relationships

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*) ? _____

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ___ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*)? Changing Situation - Separating

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*)? Changing Situation – Separating

What quantities would you choose?

- **Benchmark 5 and 10: (_____, _____) (_____, _____) (_____, _____)**

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*)? Changing Situation – Separating

What quantities would you choose?

- **Benchmark 5 and 10:   (5, 2)   (10, 3)   (20, 15)**

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*)? **Changing Situation – Separating**

What quantities would you choose?

- Benchmark 5 and 10: (5, 2)   (10, 3)   (20, 15)
- 1 or 2 More/Less: (_____, _____)   (_____, _____)   (_____, _____)

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*)? Changing Situation – Separating

What quantities would you choose?
- Benchmark 5 and 10: (5, 2)   (10, 3)   (20, 15)
- 1 or 2 More/Less: (4, 3)   (7, 6)   (13, 11)

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*)? **Changing Situation – Separating**

What quantities would you choose?

- Benchmark 5 and 10:   (5, 2)   (10, 3)   (20, 15)
- 1 or 2 More/Less: (4, 3)   (7, 6)   (13, 11)
- Parts/Whole: (_____, _____)   (_____, _____)   (_____, _____)

LET'S TRY IT!

Carolina received a package of bracelets for her birthday. There were _____ bracelets in the package. She put on ____ bracelets. How many bracelets did she leave in the package?

What's the Story (*Operation*)? <u>Changing Situation – Separating</u>

What quantities would you choose?

- Benchmark 5 and 10: (5, 2) (10, 3) (20, 15)
- 1 or 2 More/Less: (4, 3) (7, 6) (13, 11)
- Parts/Whole: (4, 2) (9, 6) (18, 12)

In Summary....

**To Use Number Sense
to Build Operations Sense Through Stories:**

- *Know the STORY*

- *ADJUST QUANTITIES to target Number Relationships*

- *Use the THREE READS Strategy to help children visualize and make sense of the story*